A mathematical model for solid liquid and mass transfer coupling and numerical simulation for hydraulic fracture in rock salt*

LIANG Weiguo[1, 2 **] and ZHAO Yangsheng[1]

(1. Mining Technology Institute, Taiyuan University of Technology, Taiyuan 030024, China; 2. Rock and Soil Mechanics Institute, Chinese Academy of Sciences, Wuhan 430071, China)

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Abstract The hydraulic fracture in rock salt is a complicated solid fluid and mass transfer coupling process. Through theoretical analysis, a solid fluid and mass transfer coupling mathematical model of hydraulic fracture in rock salt is established in this work, and numerical simulations are carried out with the model. The simulation results indicate that rock salt cracks in the typical way of wing-crack (or tensile crack) during the fracture, and the relation of fracture aperture (w) with expanding distance (x) and fracture time (t) is $w=(0.0034+0.0006t)e^{(0.0007+0.0018t)x}$. Furthermore, it has been found that both the water pressure in the crack and the expanding velocity of the crack decrease gradually as a result of the influence of salt dissolving during fracturing. These numerical simulations well illustrate the process of hydraulic fracture in rock salt and are significantly meaningful in engineering practice.

Keywords: rock salt, hydraulic fracture, solid liquid and mass transfer coupling, mathematical model, numerical simulation.

The hydraulic fracturing technique is widely used in the oil engineering, natural gas exploiting, underground heat extracting and other mineral resources mining, etc. with well developed hydraulic fracture models for ordinary rocks[1—3]. In the field of solution mining of salt deposit, this technique is used to produce fracture along the plane layer of the deposit, thus to connect the injection and target wells[4, 5]. However, due to the special physical and mechanical characteristics of rock salt, executing hydraulic fracture in rock salt deposit is different from that in ordinary rocks. Rock salt deposit is prone to being dissolved in water because of its mineral components of sodium, magnesium ion or chloride, which are easy to be decomposed and dissolved when encountered with water, and water is the very fracturing media and pressure-transferring carrier utilized in the technique of hydraulic fracture. Therefore, there is a dissolution process during hydraulic fracture in rock salt layers, which is an important influencing factor to the developing and changing of crack geometry during fracture, and consequently an important factor to be considered when establishing the mathematical model.

Normally, the fracture width is often subject to the original geo-stress and the fluid pressure in the fracture during hydraulic fracturing. However, the mineral dissolution also contributes greatly to the change of fracture width in salt rock deposit fracturing. The dissolution of the mineral in salt deposit not only affects the fracture width but also changes fluid pressure in the fracture due to the dissolution of the salt mineral, consequently the fracture propagation is affected. So it can be said that mineral dissolution is an important factor to influence the fracture propagation and the fracture geometry during the hydraulic fracturing in salt rock. Therefore, it is rational to couple the hydraulic fracturing mechanism with solution mechanism when the mathematical model of hydraulic fracturing in salt rock is established.

The fracture initiation, propagation and the fluids seepage are the main considered points for hydraulic fracturing mechanism, and mineral dissolution and fluid concentration are major for solution mechanism. The two mechanisms are coupled through the fracture width and the fluid pressure. A fracture initiates when the injected water pressure matches with the fracturing condition of the deposit at the injected well bottom, and there is a pressure gradient along the developing direction of the fracture until zero near the fracture tip. At the same time, the injected fresh water dissolves the immerged salt rock in the fracture and becomes concentrated brine, and the dissolution will not stop until the brine is saturated, but it is

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** To whom correspondence should be addressed. E-mail: master_lwg@hotmail.com

hardly saturated because of the pumped fresh water dilution continuing. With the salt dissolution continuing, the width of the fracture becomes greater and consequently the fluid pressure is rearranged along the fracture. At the tip of the fracture, the condition of the fracture propagation is the fluid pressure satisfying the demand of fracture extension according to the theory of fracture mechanics, otherwise it stops. These are the coupled mechanism of hydraulic fracturing in salt rock deposit.

Thus, hydraulic fracture in rock salt deposit is a complicated coupling process, not only for a fracturing process but also for a mineral dissolving process. The fracture aperture is often influenced simultaneously by water pressure in the crack, the original stratum stress, mineral dissolvability, and etc., among which mineral dissolvability is one of the most important factors. In fact, it plays such a great role during hydraulic fracture in rock salt that it almost dominates the distribution and change of water pressure in the fracturing crack. In this paper, a coupling mathematical model of hydraulic fracture and mineral dissolution in rock salt is established on the basis of theoretical analysis, and corresponding numerical simulations are carried out by the established model, through which some meaningful results are obtained.

1 Mathematical model of fracture and dissolution in rock salt

1.1 The equation of liquid seepage in fissure[6]

According to the equation of continuity for fluid, the seepage motion of fissure water can be described as:

$$\operatorname{div}(\rho q) = \frac{\partial(n\rho)}{\partial t}. \qquad (1)$$

In the two-dimensional situation, the constitutive equations of seepage along the plane of fissure expanding are:

$$q_1 = k_f \frac{\partial p}{\partial s_1},$$
$$q_2 = k_f \frac{\partial p}{\partial s_2}, \qquad (2)$$
$$k_f = \frac{w^3}{12\mu},$$

$$\frac{\partial(n\rho)}{\partial t} = \rho\frac{\partial n}{\partial t} + n\frac{\partial \rho}{\partial t} = \rho\frac{\partial w}{\partial t} + n\rho\frac{\partial p}{\partial t}. \qquad (3)$$

Substituting (2), (3) into (1), we obtain

$$k_f\frac{\partial^2 p}{\partial s_1^2} + k_f\frac{\partial^2 p}{\partial s_2^2} = \rho\frac{\partial w}{\partial t} + n\rho\frac{\partial p}{\partial t}, \qquad (4)$$

where p is the water pressure in the fissure, w the fracture aperture, k_f the liquid seepage coefficient in the fissure, n the void index of the fissure, and s_1, s_2 are natural coordinates of the fissure in shear direction.

1.2 The equation of rock mass deformation[7]

If the rock mass is considered as continuum media, the stress equilibrium equation denoted by displacement is

$$(\lambda + \mu)u_{j,ij} + \mu u_{i,jj} + F_i = 0, \qquad (5)$$

where λ, μ are constants of deformation property of rock mass, u is the displacement of rock mass, and F_i the bulk force.

The equation of the crack deformation resulting from hydraulic fracture is

$$\sigma_n' = K_n\varepsilon_n,$$
$$\sigma_s' = K_s\varepsilon_s, \qquad (6)$$
$$\sigma_n' = \sigma_n - p.$$

1.3 Law of the crack expanding[8]

According to the theory of fracture mechanics, the crack resulting from hydraulic fracture is a wing-type crack (or tensile crack), and the crack spread is a brittle fracture process of the tip crack. The law of the crack spread can be described by the expanding condition and brittle fracturing of crack, which is

$$K_{\mathrm{I}} \geqslant K_{\mathrm{IC}}, \qquad (7)$$

where K_{I} is stress intensity coefficient and K_{IC} the critical stress intensity coefficient of the rock mass.

1.4 The equation of salt dissolution in the fracture

Under the function of hydraulic pressure, rock salt breaks and the crack gradually expands along the mineral layer, along which the water flows and dissolves the upside and downside of rock salt crack simultaneously, which in turn results in the increase of fracture aperture. The water dissolving the salt around the crack gradually turns into chemical liquids with high concentration, and reaches equilibrium by means of two types of mass-transfer of diffusion and convection. The relation between diffusive flux and concentration of the liquid can be described by Fick's diffusive law

$$J_i = -D_{ij}\frac{\partial C}{\partial x_j}, \qquad (8)$$

where J_i is the component of diffusive flux, C the concentration of fluid changing with location and time, and D_{ij} the component of diffusive coefficient. According to the diffusive law and the conservation of mass, the equation of diffusion and convection of chemical fluid in the fracture can be derived, which is

$$\frac{\partial C}{\partial t}=\frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial C}{\partial x_j}\right)-\frac{\partial}{\partial x_i}(CV_i)+I. \qquad (9)$$

The first term on the right-hand side of Eq. (9) denotes the transfer of chemical liquid caused by diffusion; the second term denotes the transfer caused by convection of fluid between the locations of different concentration; t denotes time; I denotes the term of sources and sinks of concentration which is the function of the solubility of salt mineral, the concentration and the temperature of fluid.

In the Cartesian coordinate system, for isotropic fissure rock medium the diffusive coefficient can be described by

$$D_{ij}=\alpha_T V\delta_{ij}+(\alpha_L-\alpha_T)V_iV_j/V, \qquad (10)$$

where V is the average flow velocity of fluid field; V_i and V_j are component velocities respectively in the direction of the coordinate system; α_L and α_T are diffusion in lengthways and transverse orientation respectively; δ_{ij} is chronicle delta. Coupling the equation and boundary condition, we can obtain the mathematical model of concentration fields for salt liquid in dissolution space, that is

$$\left.\begin{aligned}&\frac{\partial C}{\partial t}=\frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial C}{\partial x_j}\right)-\frac{\partial}{\partial x_i}(CV_i)+I\\&C=C_i(x_i,t)\quad \text{on}\quad s_1\\&\left(CV_i-D_{ij}\frac{\partial C}{\partial x_j}\right)n_i=g(x_i,t)\\&D_{ij}=\alpha_T V\delta_{ij}+(\alpha_L-\alpha_T)V_iV_j/V\end{aligned}\right\}. \qquad (11)$$

1.5 The solid fluid and mass transfer coupling mathematical model

It is more reasonable to couple the solid and liquid media when analyzing the spread and dissolution of rock salt crack during the process of hydraulic fracturing, because the change of water pressure in the fissure can result in the change of normal stress of crack, and subsequently the fracture aperture is influenced; on the other hand the change of fracture aperture is controlled by the stress field and influences the water pressure in the fracture at the same time. Such a solid, fluid and mass transfer coupling mathematical model of fracture and dissolution interaction can be described by the following equations:

The equation of liquid seepage in fissure:

$$k_f\frac{\partial^2 p}{\partial s_1^2}+k_f\frac{\partial^2 p}{\partial s_2^2}=\rho\frac{\partial w}{\partial t}+n\rho\frac{\partial p}{\partial t};$$

The equation of rock mass deformation:

$$(\lambda+\mu)u_{j,ij}+\mu u_{i,jj}+F_i=0;$$

The equation of the crack deformation:

$$\sigma_n'=K_n\varepsilon_n,$$

$$\sigma_s'=K_s\varepsilon_s,$$

$$\sigma_n'=\sigma_n-p;$$

Law of the crack spread:

$$K_I\geqslant K_{IC};$$

The concentration field equation of brine in dissolution space:

$$\frac{\partial C}{\partial t}=\frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial C}{\partial x_j}\right)-\frac{\partial}{\partial x_i}(CV_i)+I.$$

2 Numerical method and simulation for the coupling mathematical model

A finite element numerical solution and the methods of coupling and iterative approach are then applied to the above coupling mathematical model. The solid deformation and stress field, which are the functions of water pressure and self-weight stress of stratum, are calculated on the time of t_i, and the length and aperture of fracture can be figured out. Taking the results into the liquid seepage equation, the flow velocity and the distribution of water pressure in the fracture can be obtained. Then take the calculated results into mass transfer equation, the fracture aperture resulting from the coupling interaction of hydraulic fracture and salt dissolution can be acquired. Substitute the water pressure and dissolving thickness into the new data in the equation of solid deformation, the expanding length and aperture of the fracture can be obtained. Such calculations are continued repeatedly and the rule of fracturing and dissolving of rock salt during hydraulic fracturing can be finally derived.

The detailed calculations are as follows: (i) calculate the time differential coefficient with Eqs. (2) and (3), meanwhile do circulation of time increment; (ii) calculate the displacement and stress of each nod at time t_0, and obtain the fracture aperture and the water pressure distribution in the crack, which is then turned into the equivalent load at each nod in rock mass; (iii) calculate the diffusive equation on the basis of the flow velocity in the fracture and the dissolving thickness of the crack during the time increment, then adjust the load on the rock mass according to the change of the dissolving thickness of rock salt by means of the initial stress method; (iv) calculate the equation of solid deformation under the load originated from the coactions of water pressure and salt dissolution and take the result of fracture aperture w into Eqs. (2) and (3), then solve the equations subsequently and substitute the parameters in the equation into new results till $|p_{t+1}-p_t|\leqslant\delta$; (v) repeat the above calculating steps and the law of crack expanding and water pressure distribution during the process of hydraulic fracture in rock salt can be obtained. The sketch map of coupled calculating is shown in Fig. 1.

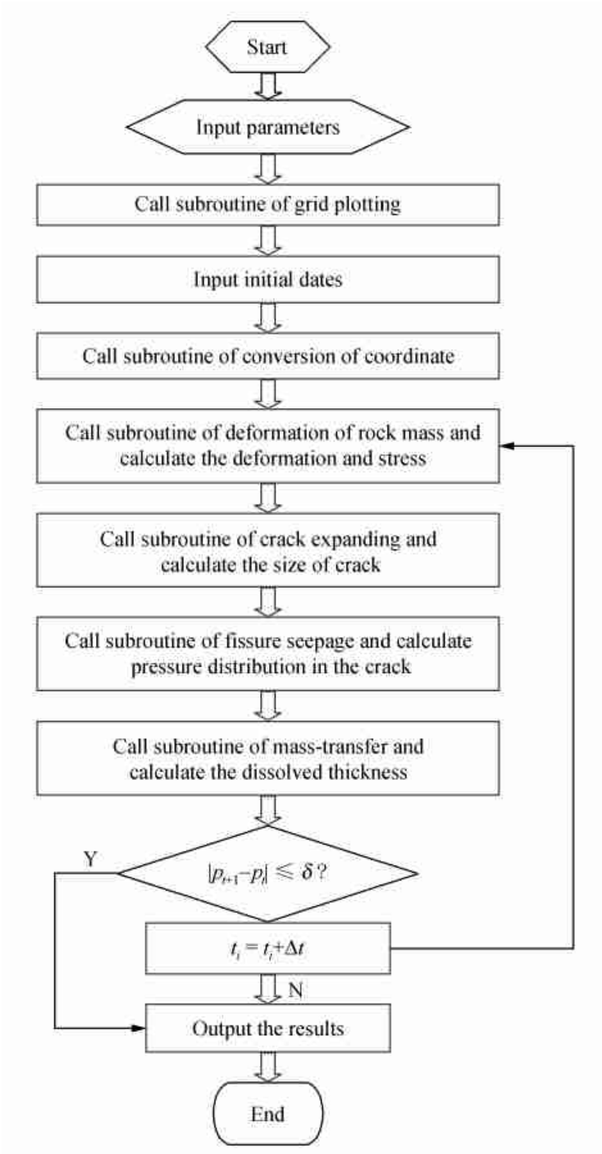

Fig. 1. Sketch map of solid fluid and mass transfer coupling numerical simulation.

Using the solid, fluid and mass transfer coupling mathematical model of fracture and dissolution in rock salt and the corresponding finite element calculating program, the numerical simulation can be carried out with respect to the physical model, as shown in Fig. 2.

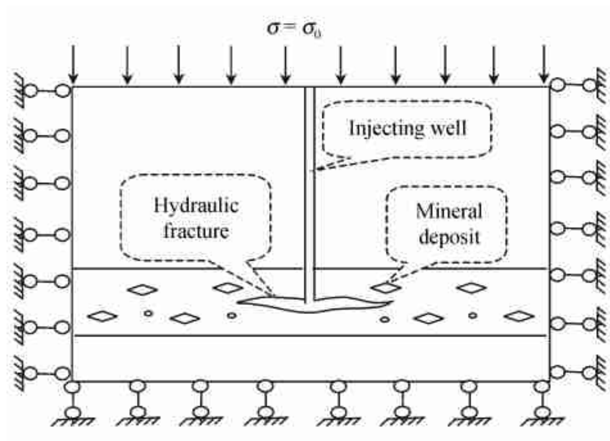

Fig. 2. A physical model of hydraulic fracture in rock salt.

The boundary conditions of physical model are as follows:

Solid space: the load on upper side is evenly distributed, $\sigma=\sigma_0$, both the left and right sides are fixed supports;

Fluid in fracture: the flux is given in drilling (or injecting) well, $q=q_0$;

Diffusive fields of brine: the concentration is given in the vicinity of drilling (or injecting) well, $C=0$.

Through finite element methods, the law of fracture aperture changing and water pressure distribution in the crack is obtained under the coupling action of fracture and dissolution. The simulated results are shown in Figs. 3 and 4.

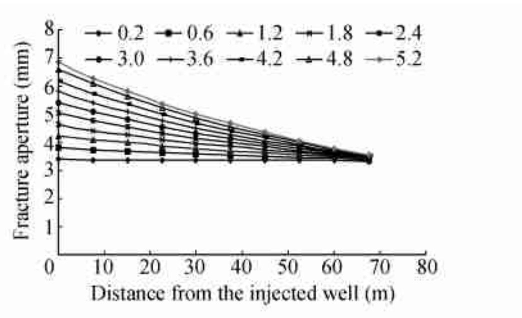

Fig. 3. The change of crack aperture with time and distance from the injected well during fracture.

Fig.3 illustrates the change of fracture aperture during hydraulic fracture in rock salt, which is the function of fracturing time and expanding length of crack. It is shown that the hydraulic fracture of rock salt has a sector-shaped opening, which is typical of wing crack or tensile crack, and the fracture aperture of the crack becomes wider with time in spite of the decrease of water pressure. After approximately 5 hours of the initial fracturing of rock salt, the fracture aperture near injecting well is 3.5 mm, which is 5 times of that when dissolving effect is ignored, as shown in Fig.5. This indicates that the dissolution of rock salt is an important factor in the opening width of fracturing crack. On the other hand, the crack becomes narrower with the increase of length from the injection well. All of these are the result of the coupling action of hydraulic fracture and salt dissolution. Through regressive analysis, the relation of fracture aperture with fracturing time and fracture length is obtained, that is $w = (0.0034 + 0.0006t) e^{(0.0007+0.0018t)x}$.

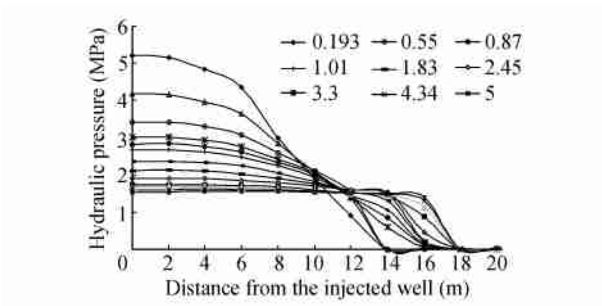

Fig. 4. The change of hydraulic pressure with time.

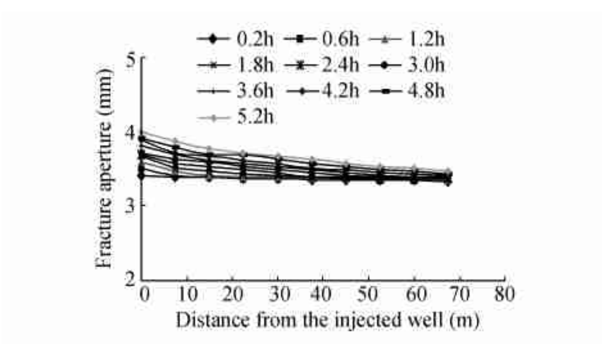

Fig. 5. The change of crack aperture with time and distance from the injected well during hydraulic fracture in ordinary rocks (without solution factor).

Fig.4 illustrates the change of water pressure distribution in rock salt fracture during hydraulic fracture, from which the development process of crack resulting from the water pressure can be obviously observed. With the fracture expand and fracture aperture change, the water pressure decreases with the crack spread till crack tip, where the stress intensity coefficient K_I no longer satisfies the law of fracturing and expanding of rock crack. Meanwhile, it can be found that the water pressure in the crack and the expanding velocity of the crack both decrease gradually because of the influence of salt dissolution, which reminds us of the necessity of gradually increasing pumping water volume during the fracturing process, so as to meet the demands of expanding stability of the crack.

The fracturing and expanding process of crack is shown in Fig.6, where it can be found that the fracture expands only if the water pressure satisfies the conditions of the law of rock fracturing and crack expanding. It is illustrated that the crack expansion can be classified into different stages, and the curve demonstrating the expanding process of fracture is consistent with the result of Shen et al.[9]. The crack develops when the conditions for the fracturing rule are satisfied, and stops when not.

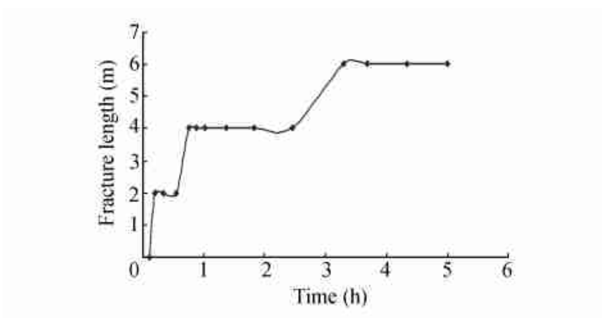

Fig. 6. The fracture expanding curve of hydraulic fracture with time in rock salt.

The above analysis takes the coupling action of fracture and dissolution in hydraulic fracture for rock salt into consideration, and a separate analysis ignoring the dissolution effect during fracturing is also carried out, the result of which is shown in Fig.5. Compared with Fig.3, the fracture aperture is obviously narrower when the influence of salt dissolving in crack is ignored.

3 *In situ* test

3.1 Geology of salt deposit in Yuncheng, China

Yuncheng salt deposit is the sediment formed during the Quaternary period, the main underground mineral resources of which are mirabilite and glauberite chemical deposits. The deposit horizontally lies in the stratum between the lower Pleistocene and

Holocene Series of Quaternary System about 69.30 m to 113.06 m underground. The main mineral for mining is mirabilite, which lies about 80 m underground and is 3.0 m thick on average. The length of the deposit is about 4.5 km in east-west direction and the width 1.6 km in south-north direction. Both the roof and the floor of the mirabilite stratum are glauberite, and the average thickness of which are 1.63 m and 1.79 m respectively.

Geological survey report indicates that the covering of the deposit is clay of Quaternary System, and the geologic structure is simple throughout the deposit area without any faults. Consequently the horizontal tectonic stress is not considered in the hydraulic fracturing design for mirabilite deposit. In addition, the drilling core samples of the minerals illustrate that there are some thin clay interlayers existing in the deposit, which is helpful to the hydraulic fracturing connection.

The mechanical parameters of the mirabilite and glauberite are shown in Table 1. It can be found that the mechanical characteristics of the mirabilite deposit are inferior to that of the glauberite deposit, which is helpful to avoid the hydraulic fracture intruding into the hard glauberite deposit when the hydraulic fracturing is carried out in the mirabilite deposit. The hydraulic fracture can easily expand forward along one of the soft clay interlayers in the mirabilite deposit.

Table 1. Mechanical characteristic parameters of the salt deposits in Yuncheng

Deposit	Compressive strength (MPa)	Tensile strength (MPa)	Cohesion (MPa)	Friction angle (°)
Mirabilite	1.2—3.1	0.2—0.4	0.6—1.1	38.1—42.2
Glauberite	13.8—35.2	1.8—2.6	6.5—7.9	18.3—22.5

3.2 Hydraulic fracturing connection in the salt deposit

Because the salt deposit is thin, hydraulic fracturing connection solution mining method was suggested as an efficient method being used in August 2002. According to the geological condition of the salt deposit, the distance between well 1 and well 2 is designed as about 45 m. After the steel tubes of the two wells are grouted with the stratum, the hydraulic fracturing connection operation between the two wells can be carried out.

Well 1 was designed as the injection well and well 2 was objection well. The fresh water was pumped into the well with BW-250 mud-pump, whose maximum ability is 7.0 MPa. The primary input of fresh water was as low as about 1—2 m^3/h, and the well head pressure on the ground gradually increased with the water continually injected from 0.0 MPa at first to 4.6 MPa about 20 minutes later, then the pressure decreased sharply and the well bottom salt rock broke, at the same time the water input increased to 4 m^3/h and the fracture propagated along the deposit layer and the two wells were successfully connected 22 hours later. During this course, the change of the injected well head pressure was recorded (Fig. 7). It can be found that the simulated results and tested results are profoundly in conformity with each other.

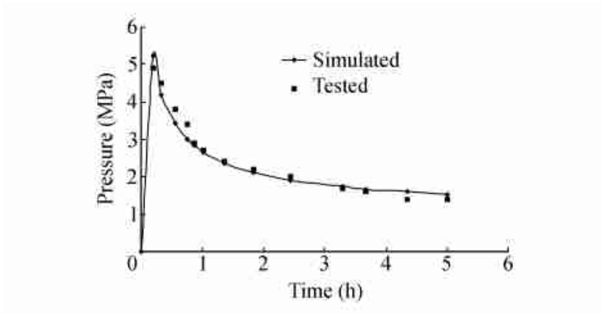

Fig. 7. Curve of simulated and tested hydraulic pressure with time in rock salt during hydraulic fracture.

4 Conclusions

(i) Due to its dissolvability in water, the hydraulic fracture in rock salt is different from that in ordinary rocks, which is a complicated coupling process that includes crack fracture and salt dissolution. The mathematical model of hydraulic fracture is established with the equation of solid deformation, the equation of fracture deformation, the liquid seepage equation, the law of fracture expanding and the solution equation.

(ii) The hydraulic crack in rock salt has a sector-shaped opening, which is typical of wing crack or tensile crack, and is obviously different from that when mineral dissolution is not be considered. Through regressive analysis, the relation of fracture aperture (w) with fracturing time (t) and fracture length (x) is obtained, that is $w = (0.0034 + 0.0006t)e^{(0.0007+0.0018t)x}$.

(iii) The pressure in the hydraulic crack gradually decreases with the crack expanding and the fracture aperture widening till the crack tip, where the pressure no longer satisfies the conditions for the law of crack expanding. Furthermore, it is found that the crack expands at different stages, and that the water pressure in the crack and the expanding velocity of crack both decrease gradually as a result of influence of salt dissolution.

(iv) The simulated results and tested results are basically in conformity with each other, demonstrating that the established solid fluid and mass transfer coupling mathematical model of hydraulic fracturing in rock salt is reasonable.

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